An Investigation into Using Applications to Facilitate the Transition from Second-Level to University Mathematics

Brian Carroll and John O’Donoghue
NCE-MSTL, University of Limerick


In recent years there has been considerable concern about the falling standards of mathematical skills of students emerging from second-level education and, in particular, of those proceeding to third-level education (Gill, 2006; NCCA, 2005; O’Donoghue, 1999). The significance of the ‘Mathematics Problem’ combined with growing concerns that higher education graduates of engineering, science, business and computing are lacking required levels of mathematical proficiency for economic development are issues of concern both in Ireland and worldwide (Lawson, 1997). There is a growing need for more students to make a successful transition from second-level mathematics education to third-level mathematics education so as improvements in quality and in retention and completion rates can be achieved in higher education courses that contain a significant mathematics component.

The need for students to develop a problem solving approach through the medium of applications and modelling is widely accepted and its merits for inclusion are widely recognised (Burkhardt, 2006; Ferrucci & Carter, 2003; Mustoe, 1992; James, 1985). The author’s research problem is to investigate the benefits of using applications to facilitate the successful transition from second-level to university mathematics. This paper outlines a teaching intervention aimed at the teaching and learning of upper second-level mathematics which involves the use of ICT. The mathematics focus is aimed at highlighting the potential of mathematics in relation to real-life contexts within a distinct modelling approach.

1. Introduction

In recent years there has been extensive awareness of the decline in the number of students undertaking undergraduate programs in Science and Engineering at third level institutes. Not alone are there fewer students than ever entering these programs, there is an increase in the number of students who drop out after their first year. Much of the problem is attributed to the students’ difficulties with mathematics in these courses. As a result considerable research on mathematics education has been carried out in an attempt to gain an insight into the factors determining the profile of students entering third level service mathematics courses (Liston, 2008).

The integration of applications and modelling in the mathematics curricula worldwide is still regarded as being in a period of transition. We have still to reach a stage of
general classification, or a settled consensus on its role in mathematics education (Niss, 1987; Burkhardt, 2006). However, it cannot be overlooked that applications and modelling have secured a grip in post-elementary curricula in most countries (Niss, 1987, Burkhardt, 2006). The need for students to develop a problem solving approach through the medium of applications and modelling is widely accepted and its merits for inclusion are widely recognised (Burkhardt, 2006; Ferrucci & Carter, 2003; James, 1985; Mustoe, 1992).

Investigating the role which technology can facilitate in alleviating the under-preparedness of students entering service mathematics courses has resulted in widespread changes in the teaching and learning of mathematics courses at third level institutes worldwide (Grouws, 1992; Holton, 2001). As expected, the technological era has had a profound impact on higher education, not only in mathematics courses, but across the spectrum of subjects offered at third level institutes. Consequently, mathematics educators have been faced with new choices and decisions regarding appropriate syllabi and effective pedagogical approaches, not only at third level but at all levels of mathematics education, from elementary through lower and upper secondary to third level. Internationally, the impact of technology on mathematics education has become an emergent domain of interest and significance while the debate on mathematics education and its stimulus on all levels of schooling has become widespread (TIMMS, 2003, PISA 2003).

Similar pedagogical issues and concerns have arisen in Irish education, where the ‘Mathematics Problem’ and issues involving the transition from school mathematics to service mathematics in Irish higher education has been highlighted (O’Donoghue, 1999; Gill, 2006). As a result, there is considerable concern about the low level of mathematical skills of students emerging from second-level education and, in particular, of those proceeding to third-level education (NCCA, 2005). The relatively poor take-up of Higher level mathematics (17% of students sitting their Leaving Certificate in 2008 completed the Higher level mathematics paper) rightly gives cause for concern, since it has implications for the follow-on study of mathematics to degree level (State Examinations Statistics, 2008).

The purpose of this paper is to present an outline of the research that has been undertaken to-date. In doing so, this paper outlines a teaching intervention aimed at the teaching and learning of upper second-level mathematics which involves the use of ICT. The mathematics focus is aimed at highlighting the potential of mathematics
in relation to real-life contexts within a distinct modelling approach. The intervention is specifically designed to make the students aware of the relevance of mathematics to their everyday lives and hence improve their interest and attitudes towards mathematics.

2. Background to the Research

2.1 Research Rationale

The postulated potential beneficial effect of using technology as an integral component of a well-defined modelling approach to the teaching and learning of mathematics at upper-secondary level was the origin of this study. Also, the study has been strongly influenced by the need to facilitate a successful transition from second-level to university mathematics in order to help address the ‘Mathematics Problem’ currently being experienced in third level institutes throughout Ireland.

The author himself had enjoyed a successful mathematics education both at second-level and university level, however, the author had noticed that not all students, in fact only a minority, could deem their mathematics education a success. This realisation, coupled with the author’s experience in teaching mathematics at upper-secondary level, led the author to question the effectiveness of traditional pedagogical approaches to mathematics at this level as he found a significant number of students struggling to learn and appreciate mathematics.

The author has personal experience of technology in the teaching of mathematics at third level. His experience of the possibilities available to mathematics students through the use of ICT in his own studies at third level contributed to his assured confidence in this medium of learning. It ensured the author became familiar with and appreciative of the potential that ICT has as a medium through which the learner can realise the role which mathematics can play in real-life contexts. Furthermore, the increasing availability of inexpensive hand-held graphic calculators and computer algebra systems has widened the range and potential of problems and models which can be treated in educational contexts (Niss, 1987; James, 1985). However, it seems that even now when they are widely available, mathematics teachers rarely use computers in their educational practise. Consequently, the author became increasingly interested in the potential contribution ICT could make in mathematics teaching and
learning through a modelling approach to enhancing the experience of upper-secondary level mathematics students. The author’s personal experiences have afforded a personal perspective on the transition from senior cycle to university mathematics from both sides, which has the potential to add insight and understanding.

2.2 Scope and Significance of the Research
The significance of the ‘Mathematics Problem’ combined with the growing concerns that higher education graduates of engineering, science, business and computing are lacking required levels of mathematical proficiency for economic development are issues of concern both in Ireland and worldwide. There is a growing need for more students to make a successful transition from second-level mathematics education to third-level mathematics education so that improvements in quality and in retention and completion rates can be achieved in higher education mathematics courses. While there has been substantial research undertaken on using applications in the teaching and learning of mathematics in both higher education and post-primary education, there is a need to examine and maximise the possible contribution of applications in both the curriculum and assessment process in the upper secondary (senior-cycle) level, thereby addressing an important aspect of the ongoing ‘Mathematics Problem’.

2.3 Limitations of the Study
The author recognises that there are some limitations to the study. Firstly the sample group used for this study will be drawn from a relatively small pool. This project is designed to examine the attitudes of students in Senior-Cycle mathematics who are preparing for third level education. Therefore, subjects not suitable for the project would include students in Junior-Cycle or those sitting Foundation level mathematics, as well as mathematics teachers not teaching the Senior-Cycle mathematics courses. The students involved must be preparing for Leaving Certificate mathematics at Higher or Ordinary level. Teachers involved must also be teaching mathematics at Senior-Cycle level. Secondly, due to nature of the senior-cycle mathematics curriculum in Ireland, there exists issues regrading time constraints. The time allotted to mathematical studies can be altered little so as to facilitate all existing and prospective aspects, thereby providing modest flexibility for change.
The time allotted to the pure elements of the mathematic curriculum would have to be diluted or reduced to allow room for applications and modelling to be facilitated (Niss, 1987; Burkhardt, 2006). Many mathematics teachers and policy makers would be unwilling to allow such dilution as it would represent to them a decline in the quality of the mathematics education provided to students, a decline not compensated by the significant mathematical understanding gained through applications and modelling activities (Niss, 1987; Burkhardt, 2006).

It is envisaged that due to these time constraints, the pool of potential participants will be restricted to those students currently in transition year. The author acknowledges this limitation. However, the author believes that, as these students are preparing for senior-cycle mathematics courses and intend to take mathematics to Leaving Certificate level, they are valid participants. These limitations should not prevent the project from contributing significantly to the solution of the ‘Mathematics Problem’ in Ireland.

3. Theoretical Framework

In an attempt to alleviate the cognitive conflict which the students are faced with in learning mathematics an analysis of a suitable theoretical framework was required. The author proposed an adaptation of APOS theory, developed, as a result of the work of Dubinsky and his colleagues in the Research in Undergraduate Mathematics Education Community (RUMEC), to suit the needs of the learners.

3.1 APOS Theory

APOS theory is, primarily, a constructivist theory that deals with the approaches an individual uses in learning mathematics. The theory has been developed, as a result of the work of Dubinsky and his colleagues in the Research in Undergraduate Mathematics Education Community (RUMEC), in a bid to expand Piaget’s concept of reflective abstraction to encompass undergraduate mathematics education (Asiala et al., 1996). It is proposed to adapt the APOS theory model in a way that the beneficial activities like cooperative learning and computer activities remain as core activities. Geogebra will be used to promote learning in a student-friendly manner in the place of computer programming. The framework has its roots in action research in that it uses feedback from data in an ongoing cyclical process (Cohen et al., 2000).
3.2 An Adaption of APOS Theory for Applicable Mathematics

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The framework has its roots in action research in that it uses feedback from data in an ongoing cyclical process (Cohen et al., 2000). There are three phases: The Exploratory Phase, the Implementation Phase and the Reflective Phase.

The purpose of the Exploratory Phase was to explore the issues involved which were highlighted through a survey of the literature and fieldwork. The fieldwork involved was based on the data collected from two separate senior cycle classes. Field data was collected in the form of: (a) questionnaires to assess the students’ attitudes towards mathematics, their attitudes towards their studies and their usual way of studying, while also exploring their views on applicable mathematics; (b) participating teacher’s reflective journals and semi-structured interviews. This phase was developed so as to ensure the initial adaptation of the research framework APOS theory was a good fit.

It is intended that in the Implementation Phase, the author will then repeat the fieldwork from the Exploratory Phase, implementing change based on the knowledge and experience gained during the Exploratory Phase. While in the Reflective Phase, the author will create a synthesis on the various themes and understandings that emerged during the previous two phases.

3.3 APOS Theory Limitations for this Research Project

Much of the research carried out by RUMEC was primarily focused on students who were studying mathematics as mathematical specialists. Cooperative learning and computer programming were activities that were promoted so as to foster the mental constructions called for by the theoretical analysis. While computer programming would be a familiar activity for the students that APOS theory was originally designed to help, the students participating in the teaching intervention undertaken in this research project would not be familiar with such an activity.
The author believes that the model can be adapted in a way that the beneficial activities like cooperative learning and computer activities remain as core activities. However, in the place of computer programming the author believes that Geogebra can be used so as to promote learning in a student-friendly manner. Subsequently, several avenues were explored during the Exploratory Phase to find a suitable replacement strategy that would foster the mental constructions called for by the theoretical analysis. This approach was consistent with the APOS theory framework.

In the remainder of this paper the author will describe the Exploratory Phase of the research highlighting appropriate key areas where vital knowledge gained gives the author valuable insights into the mathematical learning needs of the learner.

4. The Exploratory Phase

The method adopted in the Exploratory Phase is based on an extensive review of the literature, the experience of the author as a teacher of mathematics at both second and third level institutions. This attempts to find a meaningful approach for which APOS theory is consistent.

The author was faced with three initial challenges:

1. How to design a theoretical analysis of functions that would best predict how a student might engage in practical applications to real-life situations, while forming the necessary constructs during the learning period
2. To find suitable student-friendly practical applications that would foster the mental constructions called for by the theoretical analysis.
3. To find suitable methods of observation and assessment to test if the adaptation of the theory was indeed beneficial to students learning mathematics at senior cycle grade.

The author chose to analyse a specific important concept that is taught on the senior-cycle mathematics syllabus in Ireland. The primary aim in doing so was to narrow the focus of the research project. The concept chosen was that of a Function.

4.1 Theoretical Analysis of Functions for the Exploratory Phase
The theoretical analysis is a genetic decomposition of functions into actions, processes and objects. Figure 1 is an adaptation of the original illustrative representation of a theoretical analysis (Asiala et al, 1996, p.9) and shows how function actions are interiorized into function processes and these in turn are encapsulated into function objects. At later phases the function objects become function actions and the process continues in an upward spiral until a complete function object, called a function schema, is attained.

![Diagram of theoretical analysis of functions](image)

**Figure 1: Illustrative representation of a theoretical analysis of Functions**

To construct the initial theoretical analysis for the teaching of functions to senior cycle mathematics students, the author used his experience as a mathematics teacher at both second and third level and his own experience of learning mathematics, coupled with an extensive literature review of other approaches. From experience the author found that many students are unable to appreciate the role of mathematics in everyday life, where they fail to understand or realise the influence
that mathematics can exhibit in one’s future education and/or work-life. Current practices in the teaching and learning of mathematics in senior cycle schools in Ireland generally fail to make the necessary connections between mathematics and its place in real-life. As a result students have displayed an unwillingness to accept any part of a mathematical concept without understanding the reasons why certain approaches are taken, and more notably, the students must become conscious of the role these concepts can play in real-life situations.

Both the ‘how’ and ‘why’ of mathematics is a delicate and well debated topic. For the typical student the ‘how to do it?’ comes first and this is an action stage. The ‘why to do it?’ part comes about as a result of moving through the stages of process, to objects and finally to the schema stage where a full understanding is displayed. The exposure of students to mathematics that leads to genuine applications is an ideal vehicle for revealing to students the power of mathematical thought, while ensuring the students can acknowledge both the how and why of mathematical concepts.

The traditional method of introducing functions is as an introductory element of Calculus on the Higher Leaving Certificate syllabus in Ireland. Generally, it is assumed that the student is aware of the definition and classification of a typical function. By and large the significance of the topic in relation to applications is left to the individual teacher.

Direct entry to third level education from second level in Ireland is almost exclusively controlled by the Points System. This system, which is based on 6 subjects taken to Leaving Certificate, is used to allocate places to qualifying students who are ranked by the number of points accumulated by their Leaving Certificate grades. The dominance of the teach-to-the-examination approach in order to maximise points, as opposed to teaching for understanding, ensures that the study of genuine applications is minimal (Gill, 2006).

The author feels that there is a need to present the students with an environment which contains as much as possible about functions at the beginning, as opposed to being sequentially organised as is traditionally practised in mathematics classrooms, or as in many cases deemed as prerequisite knowledge.

Focusing on the concept of a function from the perspective of three distinct approaches (algebraic, graphical and numerical), and in addition, the consideration of mathematical concepts through applications before moving onto the concept of the derivative or the integral is a major change in direction from the normal pedagogical
approaches taken by traditional text books and is based on the intuition and experience of the author.

The theoretical analysis described above was the first component of the adaptation of the APOS framework in the exploratory phase. Due to the restricted time commitments of the participating schools the entire function schema was not observed. The theoretical analysis was developed by the author prior to the implementation of the intervention in the participating schools.

Following this, the instructional treatment was designed so as to correspond with the theoretical analysis providing sufficient material and exercises for the participating teachers and students. This instructional treatment was developed to explore areas where potential mathematical problems might occur and following the analysis and observation of the intervention, this knowledge could be combined with the theoretical analysis in order to develop a refined instructional treatment for the implementation phase. The instructional treatment employed in the exploratory phase will be discussed in the following section.

4.2 Instructional Treatment for the Exploratory Phase

The initial instructional treatment was developed prior to the intervention and presented to the participating teachers so as to enable the teacher’s to become familiar with the material. As already stated, time constraints for the participating teachers, was a key factor in the design of the theoretical analysis, thus ensuring the instructional treatment was developed accordingly.

To reduce the risk of error and also to save time a set of printed notes was given as a form of mini-textbook to each student. A teacher copy was also provided which had additional notes on mathematical modelling, while also including the required solutions for the real-life problems.

It was envisaged that the pace and structuring of each lesson would be determined by the participating teachers. The ACE Teaching Cycle proposed by Dubinsky and his colleagues in the Research in Undergraduate Mathematics Education Community (RUMEC) has been employed in this intervention and is expected to be undertaken where possible. The three components of the ACE Cycle are: Activities, Class Discussion, and Exercises.

The activities were centred around students working in small groups in the computer lab on computer tasks designed to foster specific mental constructions suggested by
the theoretical analysis. Furthermore, the activities were designed so as to ensure the students became familiar with mathematical modelling. The students were also provided with basic training in the use of Geogebra so as to ensure that they could utilise the graphing utility provided by the dynamic software. An influencing factor in the exclusive use of Geogebra was not only the extent of degree of user-friendliness but the fact that the software is completely free to download.

The problems explored in the mathematical modelling sessions were designed so as to supplement the instructional treatment, while providing the students with genuine applications that they can relate to. Where appropriate the problems were designed so as to reinforce the students’ appreciation and acknowledgment of the position of mathematics in their own lives as demonstrated in the classroom discussions.

The problems presented are open-ended in that there is more than one approach and more than one solution, depending on the student’s analysis of the problem. The use of an approach favouring common sense is routinely required. This is not explicitly stated in the problem but should be deduced from everyday life experiences and utilised to solve the mathematics problem.

The Classroom Discussion provided an opportunity for these same groups to work on paper and pencil tasks based on the computer activities. The teacher could also avail of the opportunity to provide definitions, explanations and overviews of the concepts being discussed and worked on. All new concepts, rules and definitions were presented (where appropriate) through the medium of an application to real-life as opposed to the traditional method of introducing new concepts, rules and definitions, then learning the algorithms and procedures associated with them while finally applying these procedures to an oft out-dated real-life problem. This procedure was employed so as to ensure the students could appreciate and acknowledge the position of mathematics in their own lives.

Geogebra applets were employed to introduce the concept of the linear and quadratic functions so as to ensure the students could understand and appreciate the graphing qualities they possess. The applets provided a visual stimulus through which the students could observe the effects that altering the coefficients of \( x \), the constant \( c \) etc. would have on the graph. Furthermore, the applets ensured the teacher could provide this stimulus quite effortlessly through the movement of a mouse as opposed to a time-consuming demonstration on the board.
The primary focus of this intervention is on conceptual understanding and it implies that where appropriate, topics should be presented graphically, numerically & statistically, and algebraically. Ultimately, the order in which these approaches are used varies, and all are regarded as important. This approach ensures the students are provided with an opportunity to appreciate the fact that often a combination of techniques are used when solving problems arising in real-life contexts, both in industry and everyday experiences.

Exercises were presented in relatively traditional fashion for students to work on. They are generally expected to be completed as homework. The problems attempt to probe the students understanding, where possible, from all three perspectives - graphically, numerically & statistically, and algebraically. Often the students are asked to explain their answers in words or graphs, an approach not traditionally employed in senior cycle mathematics in Ireland. Furthermore, there are few examples in the text that are exactly like the homework problems. This ensures the students cannot just look at a homework problem and search for a similar worked-out example. Success with the homework will come by grappling with the ideas presented.

4.3 Observations and Assessments for the Exploratory Phase

The participating teachers for the Exploratory Phase were observed and assessed as follows:

- Reflective Journals
- Follow-up Interview

The participating students for the Exploratory Phase were observed and assessed as follows:

- Questionnaires

The questionnaire was distributed to all the students taking part in the intervention from both schools prior to commencement of the intervention. To date 14 questionnaires from School A and 21 questionnaires from School B were completed in full and returned to the author. The data (both categorical and continuous) resulting form the questionnaire will be analysed using the statistical package SPSS (version 16.0).
The reflective journals were returned by both participating teachers, and a follow-up interview with Teacher B has been completed.

5. Future Work & Discussion
There is a substantial amount of work required for completion of this research project. All work completed to date in the Exploratory Phase will form the basis of the Implementation Phase. This work will be guided by the research questions for the Implementation Phase as outlined below:

- What aspects of the theoretical analysis need to be amended?
- What is the best method, from a practical perspective, to introduce new concepts, rules and definitions through applications (i.e. text-book, overheads, weblink or PowerPoint)?
- What impact has learning mathematics through applications and the modelling process on students’ understanding?
- What impact has teaching mathematics through applications and the modelling process on teacher’s attitudes towards the teaching of mathematics?

It is clear that there are real concerns affecting the teaching and learning of mathematics in senior-cycle curricula in Ireland. To ensure these concerns are alleviated, acceptable pedagogical experiences in the mathematics classroom must be available. Furthermore, there is a growing need for more students to make a successful transition from second-level mathematics education to third-level mathematics education so as improvements in quality and in retention and completion rates can be achieved in higher education mathematics courses.

There is a need to examine and maximise the possible contribution of applications in both the curriculum and assessment process in the senior-cycle level, thereby addressing an important aspect of the ongoing ‘Mathematics Problem’. Without such intervention, we will continue to provide a schooling experience that is conducive to the under-preparedness of our students entering third-level mathematics courses. Through the design, implementation and evaluation of this intervention it is anticipated that positive outcomes will be achieved. This intervention aims to provide the students with learning opportunities that allow the students to appreciate the role
of mathematics within everyday life, thus, the students can begin to realise the influence that mathematics can exhibit within one’s future education and/or work-life.

Bibliography


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