Threshold concepts and their use in the professional development of mathematics teachers: a methodology for collaboration across four European countries

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Abstract

This study is focused on the introduction of threshold concepts (Meyer and Land, 2006) to mathematics teachers in Finland, Portugal, Spain and the UK as part of their professional development. A multidisciplinary team drawn from the four participating countries developed a methodology to identify threshold concepts in the teaching of ratio. A flexible multiple case study approach with an exploratory perspective allowed the attenuation of differences arising from the contexts of each of the four countries and research teams whilst providing a basis for comparative analysis. The research design involved combining technologies such as pathfinder associative networking and knowledge mapping software, developed specifically for the research, and traditional methods.

Through this methodology, it was intended that teachers from Finland, Portugal, Spain and the UK in early secondary education would be supported in identifying threshold concepts in their teaching of ratio. Currently, the concept of 'ratio' is taught as a series of core concepts including proportion, fractions, percentages, comparison, and combining objects in all four countries, and in all four countries learners find the mastering of ratio 'troublesome' (Perkins, 1999: Hodgen, et al., 2011). Therefore 'ratio', or a component core concept of ratio, may be identified by teachers as having the characteristics of a threshold concept. We proposed that the identification of threshold concepts would help teachers address the conceptual difficulties associated with the teaching of ratio. The threshold concepts identified would then be shared with teachers across the four nations. Resources would be developed to tackle the ‘troublesome’ aspects, and be made available online as part of disseminating the research and its findings.

Although the bid narrowly missed being funded, the experiences of establishing effective communication pathways have led to co-operation and collaboration in other areas of scholarship and research. This paper highlights not only the substantive parts of the bid its overall rationale and methodology, but also offers some reflections on the process of constructing a collaborative funding bid with European partners based on teacher engagement.
Introduction

This paper evaluates the experience of co-ordinating a joint project between the University of Bedfordshire (UK), the University of Evora (Portugal), the University of Jyväskylä (Finland) and the University of Extremadura (Spain). The project, Developing Conceptual Understanding in European Mathematics Education, was submitted in early 2012 to the Comenius Multilateral Project/ Lifelong Learning Programme of the Education, Audiovisual and Culture Executive Agency (EACEA) of the European Commission (EC). The project intended to provide an opportunity for researchers and practitioners in education and associated fields to relate Threshold Concept Theory (Meyer and Land, 2006) to areas of mathematical difficulty and compare different ways that key mathematical concepts could be presented and taught. This would have involved using technology at various stages to identify and address ‘troublesome’ areas termed ‘liminal space’ (Meyer and Land, 2003; Raiker, 2010) within a known aspect of difficulty in mathematics in four EU countries. The use of emerging technologies to provide opportunities to promote trans-EU academic-practitioner collaboration and dissemination of resources and research was considered a strength of the proposal. It was intended that the project would present research-based knowledge about effective teaching and learning practices in novel forms, accessible and usable by policy makers, academics and practitioners. The project’s planned outcomes of process and product, termed the Framework, would have been made available to all EC countries as an Open Education Resources (OER). It was intended that the project would be extended through the OER after it had ended to apply the Framework to other area of known difficulty in mathematics with additional EU partners.

Although the bid narrowly missed being funded, the experiences of establishing effective communication pathways have led to co-operation and collaboration in other areas of scholarship and research. This paper highlights not only the substantive parts of the bid its overall rationale and methodology, but also offers some reflections on the process of constructing a collaborative funding bid with European partners.

Aims and objectives of the project

The 2000 Lisbon European Council identified knowledge as the key to future growth, jobs and social cohesion in the EU ‘...Education and training are a prerequisite for a fully functioning “knowledge triangle” (education – research – training)” (European Community: EC, 2007:3). However many pupils do not achieve their potential as predicted by various tests used in different countries and as demonstrated by the comparisons of performances in different countries provided via PISA which evaluates the quality, equity, and efficiency of school systems in 34 OECD member countries and 40 partner countries/economies. These EC- wide needs –to improve pupil outcomes, to base educational practice and training on research and evidence, and to ensure impact and dissemination of publicly funded research- have driven this project. This project makes the link between research and practice public through the innovative use of technology and teacher involvement throughout the project.

The overall aim of the proposed project was to establish a framework and e-resources to support European teachers and learners in overcoming conceptual difficulties in mathematics education. Children’s continuing difficulties in mathematics revealed by PISA (OECD, 2010) data in three of the partner countries; concern in maintaining and increasing children’s attainment in mathematics in Finland; recognition that research knowledge could be used more effectively in teaching;
understanding that ICT can facilitate understanding of theory/practice relationships.

The project’s objectives were to:

- identify threshold concepts with teachers affecting pupils’ competencies in ratio (ages 11-13)
- develop a framework to support teachers and learners in overcoming conceptual difficulties on ratio
- create e-resources to support teachers, teacher-educators and in-service trainers to identify and overcome conceptual difficulties in ratio
- share as open education resource with European countries

The impact envisaged within the partner countries and potentially throughout the EU was increased knowledge and understanding for educational practitioners of mathematical research, educational practitioners of effective mathematic pedagogy and practice and increased pupil attainment in mathematics.

The project’s development

International comparisons in 2000, 2003, 2006 and 2009 (OECD, 2009, 2010) have revealed that the performance of children in the UK has fallen behind those of other countries. The most recent OECD PISA survey of 65 countries (OECD, 2010) showed that our children’s performance compared with other countries continues to fall. Since the 2000 survey, the UK has slipped from 7th to 25th in reading, from 8th to 27th in mathematics and from 4th to 18th in science. However, poor pupil achievement in mathematics is also of major concerns in other countries, for example Spain and Portugal. Finland’s achievement in PISA has been consistently high, but politicians and academics are concerned that two home languages and growing immigration could be impacting negatively on results (Sahlberg, 2010). The Finns want to learn from countries that already have these issues. They also want to develop new approaches to close the gap between their learners and those from China, Korea and Singapore, the highest achieving PISA countries. So initial discussions between representatives from the four universities (Bedfordshire, Evora, Jyväskylä and Extremadura) identified that collaboration on some aspect of raising achievement in mathematics would have benefits for all.

The partnership, formed through recognition of mutual motivation to improve mathematics achievement, is supported by the findings of Anthony and Walshaw (2007) that an individual’s lack of competence in mathematics can adversely affect their life chances and their capacity to contribute fully to the economic wellbeing being of their society. However, mathematics is a wide-ranging subject. Focus was required for a meaningful and realistic research project to be devised. A paper presented by world-renowned academics from King’s College London and the University of Durham (Hodgen et al.) at the 2011 British Education Research Association conference in London in 2011 provided the focus. Their 4-year research project revealed continuing nation-wide concern by mathematics teachers of low pupil confidence and attainment ‘...in the two key areas of mathematics at Key Stage 3, algebra and ratio’ (2011:1). Pre-submission collaboration by the partnership revealed ratio to be an area of mutual mathematical difficulty and therefore forms the focus for this project. It is intended that the framework established by this project will be transferable to other areas of difficulties in mathematics and subjects dependent on mathematics
such as the sciences and technology.

A second stimulus was that, in the UK, recent research has shown that improving education systems has priority with a number of governments (OECD, 2009, 2010; Barber and Mourshed, 2007). These reports indicate that one of the biggest influences on student learning is the quality and effectiveness of teachers (OECD, 2009). Furthermore, the White Paper *The Importance of Teaching* (DfE, 2010:5), states that ‘All the evidence from different education systems around the world shows that the most important factor in determining how well children do is the quality of teachers and teaching’. Researchers reinforce this view stating: “...the quality of an education system cannot exceed the quality of its teachers” (Barber and Mourshed, 2007:13). Therefore it is important for educational systems and the teachers within them to continue to develop their professional knowledge through continual professional development (CPD). An intention of the project was that teachers should be supported in relating interventions to the research on which they are based by presenting the theory-practice relationships in a meaningful way. This approach resonated with recent calls for knowledge that is already in existence to be used more effectively to improve these education systems (OCED, 2010), in the UK to increase access to existing knowledge about education so that they will impact on practice (Pollard and Oancea, 2010). Hence an important research area is the use and leverage of knowledge already in existence and how it can be transformed (Foley and Hargeaves, 2003; Pollard, 2008).

A further stimulus was the knowledge and understanding gained by researchers and academics in the four universities on how ICT can create communities of practice to develop the relationship between theory and practice (Wenger *et al*., 2009). There is value in teachers engaging in CPD delivered by lecturers within their own schools or local university, with teachers contributing social-constructively to the ‘communal pot’ of expertise and taking away their enhanced portion for future practice. However, this has the effect of creating silos of knowledge. The project was designed to explore the notion that improved or enhanced quality in CPD could be achieved more effectively by teachers coming together, not only from individual schools or universities to a central point but also by engaging at the same time online with educators in other areas of the country and countries abroad. In this way, the elements (or concepts) of mathematical knowledge underpinning understanding of ratio would be identified by teachers working with researchers. The elements of knowledge are then entered into computer software known as Pathfinder Associate Networks (PFNETs). PFNETs are used as a way of representing complex knowledge. Essentially PFNETs are a means of externalising current states of knowledge through representations. These representations take the form of arrays of related nodes that break down and map out the underlying concepts on which understanding of ratio is constructed (Figure 1). They also reveal the existence of threshold concepts (see Theoretic framework of the project below) and areas of mathematical difficulty. These maps will be used as a first point of diagnosis in a known area of mathematical difficulty- ratio.
The second point of diagnosis and the means of addressing the misconceptions and misunderstandings revealed are the online Knowledge Maps (KMs). One of the authors, researching in this field for his doctorate, would then construct Knowledge Maps (KM) from the identified elements. The content of KMs is presented as a series of layers accessed via hyperlinks. The top layer (Figure 2) provides simple explanations of the concepts identified as underpinning ratio in the PFNETs. The next layer provides, per concept, a summary of related research or scholarship associated with it. From this layer there are two links per concept. One leads to a layer on which are uploaded full research and/or scholarly papers, articles and/or presentations on the concept; the second goes to a layer populated by appropriate teaching resources/strategies. In this way complex professional knowledge such as content knowledge or pedagogical knowledge (Shulman, 1986, 1987) can be made accessible to busy professionals.

In summary, Pathway Associate Networks (PFNETs) and Knowledge Map (KMs) software were to be designed and specifically developed for identification of conceptual difficulties in mathematics. In the KMs, practice and theory would be related. The process of populating the KMs would reveal areas for future practitioner development and research as the quality of and gaps in pedagogy and research surrounding the learning and teaching of ratio and its supporting concepts were identified. Both PFNETs and KMs were to be developed by academics and IT specialists working with practitioners to ensure authenticity and validity in the eyes of the end-users, and to promote
academic-practitioner collaboration. All deliverables from the project are presented in the four partner languages. It was also envisaged that outcomes from the research could be uploaded onto the web as open education resources (OERS). The collaborating academics felt confident that they would be able to identify a framework predicated on technology because of their involvement with research and development projects using ICT in education since 1995. Interestingly it was the over-reliance and over-complication of the project’s dissemination strategy on technology that were at the root of its downfall.

Figure 2: Example of a Knowledge Map for ratio

**Theoretic framework underpinning the project**

Pedagogical strategies arise from the understanding and application of troublesome characteristic of Meyer and Land’s Threshold Concept theory (2006). Meyer and Land distinguish between a concept, which could be a step increase in knowledge and understanding, and a threshold concept which gives access to transformed understanding as defined above. Transformation is one of five characteristics of a threshold concept. Threshold concepts may also irreversible, integrative, bounded and troublesome. Threshold concepts are often ‘troublesome’ (Perkins, 1999) because they are difficult to comprehend, can be counter-intuitive and may take time for understanding to
develop. An example in mathematics is ratio, the focus of this project. It is proposed that learners (teachers and trainee teachers are regarded as learners too) who have difficulties with ratio are ‘stuck’ in liminal space, between the known landscape of knowledge and the new knowledge that is being taught. If they are not scaffolded (Bruner, 1996) through the area of difficulty and simply leave it to proceed with the next topic in the mathematics syllabus, they will in time try to use the troublesome knowledge to access new mathematical knowledge which in its turn will become troublesome.

It was intended that the proposed project would use Threshold Concept Theory to identify and address children’s difficulties in areas of mathematics that research has demonstrated cause difficulties for learners in early secondary education. PISA data collection involves 15 year olds. A recent review (ERSC, 2009) of literature of student engagement with science, technology and the mathematics on which these are based found that engaging learners in these subjects became increasingly difficult after the age of 14, and that support strategies and resources to counter disengagement needed to be established in the upper primary and early secondary years. Focusing on diagnostic and intervention strategies in mathematics to support teachers working with children in the age ranges prior to the PISA test age would therefore be helpful, not only for achievement in PISA but in raising competence in an essential life skill prior to the end of statutory schooling. Most research into threshold concepts has taken place in higher education (Thornton et al., 2010). This project aims to widen the debate to early secondary mathematics.

Meyer and Land distinguish between a concept, which could be a step increase in knowledge and understanding, and a threshold concept which gives access to transformed understanding. Threshold concepts are ‘...akin to a portal opening up a new and previously inaccessible way of thinking about something...’ (Meyer and Land, 2003:5). Transformation is one of five characteristics of a threshold concept in the sense that ‘... once understood, its potential effect on student learning and behaviour, is to occasion a significant shift in the perception of a subject or part thereof....and a shift in perspective may lead to a transformation of personal identity’ (ibid). Threshold concepts are probably Irreversible as ‘...the change of perspective occasioned by acquisition of a threshold concept is unlikely to be forgotten, or will be unlearned only by considerable effort” (ibid). Meyer and Land argue that threshold concepts are integrative as they connect areas of previous knowledge and at the same time reveal a hitherto unrecognised relatedness. They also speculate that threshold concepts are possibly bounded ‘...in that any conceptual space will have terminal frontiers’ (ibid).

Of importance to this project was the notion of threshold concepts being potentially troublesome because they are difficult to understand, the knowledge being learnt appearing counter-intuitive or alien, as though it emanated from unrecognised, even unknown, semiotic sources (Perkins, 1999).

It also may take time for understanding of threshold concepts to develop because they are fundamental to several aspects of learning, in this case of mathematics. An example is proportion, a concept underpinning multiplication, fractions, decimals, percentages, scaling and ratio.

The introduction of threshold concepts being ‘troublesome’ can be regarded as positive in that a learner’s existing knowledge is thrown into a state of disequilibrium which has to be addressed. The resulting questioning and reconceptualising of knowledge is essential for learning, for learners to pass into a transformed understanding of his/her landscape of knowledge and understanding.
However, threshold concepts can be troublesome (Perkins, 1999; Meyer and Land, 2003) because learners who have difficulties with a threshold concept become ‘stuck’ in liminal space. This is the area between the known landscape of knowledge and the new knowledge that is being taught. Learners know that the new knowledge is there to be learnt because their teachers have presented it to them; what is more teachers relate the new knowledge in some way to what the learners already know. So learners cannot go back to reside in the landscape that existed before the new knowledge came along; neither can they go forward with the new knowledge to transform their disciplinary landscape which, for the purposes of the proposed project, is early secondary mathematics. If they are not scaffolded through the area of difficulty and simply leave it to proceed with the next topic in the mathematics syllabus, they will in time try to use the troublesome knowledge to access new mathematical knowledge which in its turn will become troublesome etc., etc. Figure 1 is a representation of this process focused on ratio, an area of mathematics known to be troublesome (Raiker, 2010, Hodgen et al., 2011):

![Figure 3: Liminal space](image)

It is proposed that the identification of learners stuck in liminal space so that their mental blocks can be overcome is a fundamental aspect of the mathematics teachers’ work. Pathfinder Associate Networks (PFNET) software is seen a useful tool in both identifying threshold concepts (TCs) and liminal space (LS) for individual learners and to develop theoretical understanding and application of threshold concept theory.

**Methodology**

As a starting point, the insights gained from literature on concept mapping, threshold concept theory, ratio and its underpinning concepts, including research conducted by the partner members and experts around the world, were expanded by extending the review. As an outcome of the literature review, it was decided that qualified and student teachers would be invited to workshops and seminars in each of the partner countries to work with the core project team as co-researchers to create a collection of concepts, resources and strategies associated with the teaching/learning of ratio, a topic of known difficulty. These were to be used to populate the PFNET software (used to identify threshold concepts) and the Knowledge Map online material, the latter being linked to annotated research to encourage educators to develop theory-practice insights. All resources will be translated into the languages of the four partners.
The approach to be adopted was multiple (in this case, four) case studies (Yin, 2003). Therefore all partners would follow the same protocols using the same resources when collecting data to allow critical analyses within partners and across partners. Each partner will have a coordinator responsible for assessing project activities against milestones. Measurable indicators were built into the design in the form of deliverables.

Before the data collection there was to be a meeting in Jyväskylä with a member from the project staff from each country attending where the data collection and analysis procedures are discussed and it is ensured that the research protocols will be followed in each county. After the data collection and analysis, there would be a second meeting in Bedford UK where researchers would present a summary in English of the data collection and analysis in each country. In this meeting, a representative from each country will present findings. The aim of the seminar would be to bring together and synthesise findings into an agreed draft of the final report.

Data would have included video recordings of lessons, written tests, PFNET activities, online explanations, teacher interviews and student interviews. In each country four teachers will participate in the study. The protocol to be established in each country was to cover the following:

1. A pre-test for the pupils in the four participating classes in each country on the concepts related to the ratio. The test would be taken from questions on ratio and associated concepts appearing in the PFNET software from past PISA events to ensure standardisation.

2. After the teachers were introduced to the project and to the resources, each teacher would teach the topic of ratio. All lesson plans would be collected as data. All lessons would be video recorded. In each lesson one camera would follow the teacher who would have a wireless microphone. Another camera would follow a focus group of pupils who would also have a wireless microphone.

3. There would be an intermediate test with items corresponding to the pre-test. Pupils would also create PFNET maps and complete online explanations.

4. Teachers and researchers would identify pupils who had difficulties in learning the ratio on the basis of the lessons, tests and PFNET map activity. At this point, the teachers and the researchers in each country would have an online meeting where theories for the sources of pupils’ difficulties and ways of supporting pupils would be discussed. Particularly, the aim was to identify threshold concepts and liminal space in learning ratio.

5. Teachers would use the Knowledge Map to support the pupils in addressing misconceptions and misunderstandings identified through analysis of the PFNETS, online explanations and ex-PISA test results. Also these lessons would be video recorded. Pupils attending Knowledge Map sessions would participate in a pupil focus group interview.

6. There would be a post-test with corresponding items to the pre-test and pupils would create PFNET maps and complete online explanations.

7. The empirical research would be completed with two sets of interviews: task-based interviews (Goldin, 1998) for the pupils who had difficulties but who had been supported
by the teacher; semi-structured interviews with the participating teachers. The teacher interviews include a part where the teachers would be shown a video clip from their lesson where they were using the Knowledge Map to support the pupils with difficulties (stimulate recall interview).

All the steps had milestones attached to them and were measurable. Written tests, PFNET activities and online explanations would be analysed quantitatively to provide findings on learning progress. Also the data would be analysed to identify threshold concepts by analysing variation of and connection between the underpinning concepts of ratio.

The lesson videos focused on the teachers would be analysed by partners in their own languages to identify critical episodes (Powell et al., 2003) where the teachers were using the Knowledge Map or other resources to support the pupils’ learning. Similarly, from the student videos partners would identify critical episodes where the pupils were either having difficulties in understanding ratio or were making progress. The critical episodes would be analysed qualitatively by partners to understand ways of pupils’ learning and how it could be promoted. The data from the task-based interviews of the pupils would be analysed by partners to further understand how pupils’ threshold concepts became established or why pupils got stuck in liminal space. The interviews of the teachers would be analysed by partners to understand how teachers view the use of the Knowledge Map in supporting the pupils.

At the end of the analysis period, a member of each partner’s project team would bring his/her university’s findings in the form of a presentation and report in English to a seminar in Bedford. Syntheses from this meeting would inform the project’s final report.

**Reflections on process of developing the bid and it proving unsuccessful**

It would have been very difficult to orchestrate the project had the collaborators been totally unknown to each other at the start of creating the project. However there were several partnerships with shared constituency. The key relationship behind the collaboration was that between senior academics at the Universities of Bedfordshire and Evora. Their starting point was that, through the engagement of new researchers on the project, there would be a knowledge transfer element in addition to the outcomes specified in the proposal. The first author, the mathematics and language specialist, had worked with the senior Bedfordshire academic on various school, university and government funded projects over fifteen years. The relationship between the first author and the Finnish members of the project team had been established more recently but had developed rapidly through a shared professional interest in philosophy, pedagogy and teacher education. The authors of this paper had established a relationship through work on the application of threshold concept theory to online pathways. The Extremadura involvement arose through the cooperation developed over several years between the Universities of Evora and Extremadura in doctoral training and various European projects, not only in the area of mathematics, but in several others, particularly in new technologies and education. So networks linking the collaborators existed before the onset of the project. Relationships were strengthened by visits of the authors to Evora, of the first author to Jyväskylä and of Spanish colleagues to Evora. These face-to-face meetings were invaluable. There was time in the university, in schools, over dinner, to ensure that nuances of language were understood and particular issues arising from political-social-historical-economic-ethical considerations were appreciated. Because of these face-
to-face meetings regular Skype meetings with Jyväskylä and Evora, timetabled in a project timeline, were used effectively and empathetically. The fact that all collaborators spoke English was a factor that quickened the pace of putting the project together.

Despite established links and a common language for the project, there was the problem of coming to a shared and agreed understanding of the aims and objectives, and how they were to be achieved. Not only do colleagues need to have some ownership of the project but the design of the project may need to be flexible enough to allow colleagues to contribute to the design. On the other hand, there had to be sufficient structure and content when Raiker and Procter approached Evora, Jyväskylä and Extremedora for their involvement to be substance on which to base discussions. It was only through meeting colleagues in Portugal and presenting an incomplete but theoretically robust project to them that the potential of Pathfinder Networks emerged and were integrated into the project. Allowing the project design some flexibility gave Portuguese and Spanish colleagues ownership of an area of the project. An added bonus was that the PFNETS provided a way of visually mapping threshold concepts. The differences in these visualisations illustrated variations in how threshold concepts were perceived by different teachers and their students.

The inclusion of PFNETs in the design caused some difficulties with conceptions of overall project management. This was eventually resolved, but the emergence of this problem strengthened the realisation of the differing drivers behind the four countries and their representatives. A delicate balance has to be found between individual goals and perceptions of value and the over-arching requirements of the project; yet another instance of compromise between the personal and the social.

Within this idea of getting 'everyone on board' and flexible research designs, it was also realised that it is equally important for colleagues to share the same ontological and epistemological position. Interestingly, this did not appear to be an issue. Perhaps because of the excellent English spoken and understood by colleagues from Portugal, Finland and Spain, the English collaborators were able to convey an ontological position based on Heidegger’s *desein* (being-in-the-world), Wenger’s communities of practice and Vygotsky’s socio-constructivism. However, it can be assumed that, as these thinkers have international standing, non-English collaborators had read and studied them in their own languages and the names themselves were sufficient to transfer meaning. In addition, all collaborators were academics in the discipline of education, another unifying factor. These unifying factors have survived, despite the failure of the bid. The two authors are continuing to research and write together and with the Finnish collaborators. The Portuguese partners are continuing to work on the PFNETs with Extremedora. The second author is working with senior Bedfordshire academics on online pathways, and there are plans to involve Evora and Jyväskylä with Bedfordshire to strengthen globalisation elements of teacher education in the three countries.

However, one lesson that has been learnt is that the more countries involved in a project, the simpler the research design has to be. In some ways, the parameters of the project were set by the group’s expertise in technology. In retrospect, some of the resources that were to be bought and stored in technology could have been created more simply. For example, the concepts underlying ratio could have been created on card, or with 3 dimensional objects, and photographs taken as records for analysis. The OER of resources and exemplars in four different languages, to which
other EU countries could contribute in home language, was a worthy ambition and met the EC’s criterion of taking contributors’ languages into account when constructing the project. However, the extent of translation was expensive.

In conclusion it can be said that the experience of working in a collaboration of four countries was worthwhile. The similarities and agreements between the collaborators were far greater than the differences and difficulties. The fact that ratio is an issue for children in four countries with differing PISA outcomes supports this. The collaborators are now looking for a project for funding that will enable further trans-national research.

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