The development of a framework for understanding mathematics enrichment: a case study of initiatives in the UK

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Abstract This paper reports on a study which developed, applied, and refined a conceptual framework for understanding conceptions and practices of ‘mathematics enrichment’ in the UK. Through synthesis of available literature and case-study research focused on exploring the ideas and experiences of enrichment held by the staff organizing and students participating in four distinctive enrichment initiatives, the study identified four prototypes for ‘mathematics enrichment’. These focused respectively on the development of mathematical talent, popular contextualization of mathematics, enhancement of mathematical proficiency and learning processes, and outreach to the ‘mathematically underprivileged’. Enrichment was also found to be a product of context-dependent interactions between the people and institutions involved. The resulting forms of enrichment differed not only in conception and delivery, but also in the nature and process of the negotiation which shaped their provision.

Keywords Mathematics enrichment, Gifted education, Summer schools, Masterclasses, Mathematics competitions, University outreach

1 Background

Students’ participation in, and attitudes to, mathematics have received much public attention in the United Kingdom (UK) in recent years: many are reported to be disaffected with school mathematics (Smith, 2004); numbers continuing to study mathematics beyond the end of compulsory schooling have fallen between 1996 and 2007, in line with longer-term decline across the wider science fields (The Royal Society, 2008). In the most recent European Union (EU) survey on attitude to ‘science’ among young people aged 15 to 25 (European Commission, 2008), mathematics was found to be the least popular discipline, even among the minority of young people who would ‘consider’ studying ‘science’ subjects; interest in studying mathematics in the UK was among the lowest in EU Member States.

Over the same period, the number and variety of ‘enrichment’ initiatives, aimed at engaging young people in mathematics in general and at supporting and extending the mathematics learning of specific groups of students above and beyond the provision prescribed by the school mathematics curriculum, have increased dramatically. Such ‘enrichment’ ranges from in-school provisions delivered by teachers during mathematics lessons to out-of-school interventions sponsored and run by external agencies and institutions. Examples include:

- efforts by individual teachers or school mathematics departments to enrich mathematics lessons by incorporating stimulating tasks that provide rich opportunities for mathematical exploration and problem-solving into ‘everyday’ teaching and learning;
- school-based mathematics clubs run by teachers as extracurricular activities to supplement and enhance students’ learning;
• local and national competitions to stimulate interest in mathematics in large numbers of students and to identify and cultivate talent;
• summer study programmes, popular lectures, workshops and masterclasses to engage students in mathematics and to expand their horizon; and
• outreach and ‘widening participation’ programmes organized by higher-education institutions to attract students to study mathematics-related courses and to recruit from social groups currently under-represented in higher education or in mathematics-related studies and careers.

Despite the increasing amount of activity taking place under the ‘enrichment’ label, the concept is under-theorized and the area under-researched. The term ‘enrichment’ is often used common-sensically, masking the range of programmes that operate under its label (Feng, 2005); outcomes of enrichment are also not well-understood. The large numbers of enrichment initiatives — not just in mathematics, but across STEM (Science, Technology, Engineering, Mathematics) fields — and their role and impact in promoting the STEM disciplines to young people are currently receiving increased attention from UK policymakers (Department for Business, Innovation and Skills, 2010a, 2010b; Department for Education and Skills, 2004; Department for Education and Skills & Department for Trade and Industry, 2006; Department for Innovation, Universities and Skills, 2008; Smith, 2004).

This paper reports on a study that developed, applied, and refined a framework for understanding initiatives in mathematics commonly referred to as ‘enrichment’ in the UK. As a starting point, enrichment was taken to be a social phenomenon (Moscovici, 1984, 2000) that can be represented and operationalized in different ways by its various proponents. One aim of the study was to document, and to draw to readers’ attention, the range of rationale and practices (at a programme level) presented as enrichment, without endorsing or privileging any particular conception or form of practice. Through surveying and reporting current enrichment conceptions and practices in this way, the study also aimed to raise questions and to generate discussion about the growing enrichment phenomenon, its outcomes, and its relationship with ‘formal’ schooling — both in the UK context and however enrichment might be broadly interpreted in other parts of the world. To these ends, the study mapped current thinking about the mathematics enrichment field using a similar approach to that taken by Lerman, Xu, and Tsatsaroni (2002) for mapping the mathematics education research field, and subsequently, by Adler, Ball, Krainer, Lin, and Novotna (2005) for mapping the mathematics teacher education research field. The framework developed is grounded in literature from the mathematics enrichment professional field, and is refined through case-study research that drew on empirical data supplied by enrichment participants and by leading practitioners from the UK mathematics-enrichment community.

2 Developing a framework for understanding mathematics enrichment

Although all enrichment efforts generally aim to provide high-quality educational experiences, there are different views as to what such provision would entail, what ultimate goals might be achieved, and which audiences should be targeted. With regards enrichment initiatives in mathematics in particular, four prototypes can be delineated from the literature (Feng, 2006), each with its own distinctive espoused provision, goals and target audience. These prototypes are characterized by this study, respectively, as:

1. development of mathematical talent;
2. popular contextualization of mathematics;
3. enhancement of mathematical proficiency and learning processes; and
4. outreach to the ‘mathematically underprivileged’.
Collectively, the four prototypes formed a framework for understanding enrichment initiatives in mathematics that underpinned subsequent research.

Before confronting this framework with empirical data and subjecting it to further analysis, the four prototypes are outlined in turn below. This serves to present ideas from the literature in a coherent manner, and also to demonstrate the initial theorization from the literature that formed the first step towards developing a framework for understanding mathematics enrichment in the UK.

2.1 Type 1: Development of mathematical talent

Where enrichment focuses on the development of mathematical talent (e.g. as in Clendening & Davies, 1980, 1983; Meister & Odell, 1979; Roberts, 2005; Worcester, 1979), its proponents’ stated concerns are to identify ‘mathematically-gifted’ students, to meet their distinctive academic needs, and to advance their already notable talent in the subject, so as to cultivate an elite group of individuals for leadership positions. As a key indicator of talent, high attainment in mathematics is often treated by the proponents of this form of enrichment as a prerequisite for participation in enrichment programmes.

Substantively, enrichment of this kind may introduce additional or more difficult topics not covered by the school curriculum (for the students’ age group) or involve more advanced treatment of curriculum topics. Importance is frequently attached to matching enrichment provision to students’ interests; oftentimes, students are also given considerable freedom to pursue their own enquiry. In so doing, such enrichment seeks to address its proponents’ stated goals of helping ‘mathematically-gifted’ students find fulfilment in mathematics, and of recruiting, training, and retaining such students in mathematics and related fields.

2.2 Type 2: Popular contextualization of mathematics

Where enrichment is conceived as the popular contextualization of mathematics (e.g. by Fox, 1979; Zeeman, 1990), its proponents advocate presenting powerful — sometimes, technical and advanced — mathematical topics to the lay person in terms that are intended to render the material accessible and more readily comprehensible. The stated aim is to translate such concepts into common consciousness, to expand the audience’s mathematical horizons and experience, and to raise public understanding of mathematics and its applications. In so doing, proponents of such enrichment reportedly hope to demonstrate the importance and fascinating nature of mathematics, and to overturn negative stereotypes surrounding the subject. As such, not only can all students benefit from enrichment, as many as possible should have the opportunity to take part.

Substantively, enrichment of this kind may involve novel presentations or extended explorations of topics encountered in the school curriculum, draw on topics which may not appear at first to be linked to mathematics, or introduce accessible topics not covered by the school curriculum. Applications of mathematical ideas in ‘daily life’ and in other areas of study, and the relevance of mathematical concepts for understanding contemporary events and issues may also be emphasized.

2.3 Type 3: Enhancement of mathematical proficiency and learning processes

Where enrichment focuses on enhancing mathematical proficiency and learning processes (e.g. drawing on Beetlestone, 1998; Piggott, 2004; Rodd, 1999), it is often presented by its proponents as an informed approach to teaching and learning, such as should form an integral part of education for all students. In this light, enrichment
is an ongoing process that should pervade all aspects of teaching and learning, and should not be distinguished purposely from ‘everyday’ good practice.

Substantively, enrichment of this form may feature mathematical problems and activities that call for creative applications of mathematical ideas and techniques normally taught in school lessons. To support a wide range of students to engage in such problems and activities, appropriate teacher mediation and differentiation of content may also be emphasized. The stated goals of such enrichment are to present all students with a stimulating experience of mathematics, and to promote and foster mathematical thinking in problem-solving situations.

2.4 Type 4: Outreach to the ‘mathematically underprivileged’

Where enrichment is conceived as outreach to the ‘mathematically underprivileged’ (e.g. evidenced in Bouie, 2007; Walton, 1995), the reported motivation and commitment of its proponents are to ‘increase access’ to learning opportunities and to ‘raise engagement’ in mathematics among wider student audiences. Given the mathematical underpinnings in many of society’s social and scientific activities, enrichment of this kind is often advocated by its proponents as a means of enabling ‘underprivileged’ students to overcome barriers of engagement and to gain access to valuable opportunities in life. Efforts are often aligned with concerns for social justice and equity in the mathematics-education community. Examples of such enrichment include initiatives (with broadly-conceived ‘mathematics’ components) targeted at students from social groups that are under-represented in mathematics and mathematics-related higher-education courses and careers (e.g. girls, students from certain ethnic-minority groups, students from socio-economically disadvantaged backgrounds). A key goal of such enrichment is to ‘raise’ the students’ sense of their own capability (including in, though not limited to, mathematics and participation in higher education).

The framework for mathematics enrichment, consisting of the four prototypes outlined above, is summarized in Table 1.
Table 1 Summary of Framework for Mathematics Enrichment

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Goals</th>
<th>Target audience</th>
<th>Provision</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Development of mathematical talent</td>
<td>- Identify and develop mathematical talent</td>
<td>Mathematically-gifted</td>
<td>- Introduce additional/more difficult topics not taught in school</td>
</tr>
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<td></td>
<td>- Meet distinctive academic needs</td>
<td>and high-attaining students</td>
<td>- Involve more advanced treatment of curriculum topics</td>
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<td></td>
<td>- Cultivate elite group for leadership positions</td>
<td></td>
<td>- Material matched to students’ interests and talent</td>
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<td></td>
<td>- Help students find fulfilment in mathematics</td>
<td></td>
<td>- Students given freedom to pursue their own enquiry</td>
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<td></td>
<td>- Recruit, train and retain mathematically-gifted students in</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>mathematics and related fields</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Popular contextualization of mathematics</td>
<td>- Present powerful mathematical concepts in accessible terms</td>
<td>All students</td>
<td>- Involve novel presentations/extended explorations of school topics</td>
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<td></td>
<td>- Translate powerful concepts into common consciousness</td>
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<td>- Draw on topics which may not appear to be linked to mathematics</td>
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<td></td>
<td>- Expand students’ mathematical horizons and experience</td>
<td></td>
<td>- Introduce accessible topics not covered by the curriculum</td>
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<td></td>
<td>- Raise public understanding of mathematics and its applications</td>
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<td>- Highlight mathematical applications and relevance of mathematics</td>
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<td></td>
<td>- Demonstrate importance and interest of mathematics</td>
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<td>in ‘daily life’</td>
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<td></td>
<td>- Overturn negative stereotypes</td>
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<tr>
<td>3. Enhancement of mathematical proficiency &amp;</td>
<td>- Provide stimulating experience of mathematics</td>
<td>All students</td>
<td>- Feature mathematical problems/activities requiring creative applications</td>
</tr>
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<td>learning processes</td>
<td>- Promote and foster mathematical thinking in problem-solving</td>
<td></td>
<td>of taught ideas/techniques</td>
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<td></td>
<td>situations</td>
<td></td>
<td>- Emphasize appropriate content differentiation and teacher education</td>
</tr>
<tr>
<td>4. Outreach to the mathematically</td>
<td>- Increase access to mathematics learning opportunities</td>
<td>Students from social</td>
<td>- Integral and ongoing part of education not to be distinguished from</td>
</tr>
<tr>
<td>underprivileged</td>
<td>- Raise engagement in mathematics among wider audiences</td>
<td>groups under-represented in</td>
<td>‘everyday’ teaching and learning</td>
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<tr>
<td></td>
<td>- Enable underprivileged students to overcome barriers of</td>
<td>mathematics and mathematics-</td>
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<td></td>
<td>engagement and gain access to valuable opportunities in life</td>
<td>related higher-education</td>
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<td></td>
<td>- Raise students’ sense of their own capability (including in, but not</td>
<td>courses and careers</td>
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<td>limited to, mathematics and participation in higher-education)</td>
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3 Refining the framework: Research design and methods

Following the synthesis of the four prototypes as outlined above, which, together, constituted the framework for understanding enrichment initiatives in mathematics, the study turned to refining the framework through empirical work, so as to anchor the framework on current conceptions and practices of mathematics enrichment in the UK. To this end, the framework was confronted with empirical data through being applied in four case studies (Bassey, 1999), each focusing on a distinctive enrichment initiative in the UK, and collectively chosen as being representative of different forms of ‘good practice’ as recognized by the (mathematics) education or wider community (e.g. through awards for contribution to the communities served by the initiative, recognition by the UK Office for Standards in Education (2005), or recommendation in mathematics-education public-inquiry reports (Smith, 2004)). The initiatives thus selected were:

- NRICH Mathematics Outreach Workshops (chosen as professing characteristics of Types 3 and 4): a year-long outreach programme, composed of weekly, after-school classes, targeted at students from a disadvantaged, inner-city London borough, held in a university local to the students’ schools, and run in collaboration with NRICH;
- NAGTY Mathematics Summer Schools (chosen as professing characteristics of Type 1): a set of residential summer schools for ‘gifted’ students (defined by their performance, or potential to perform, in the top 5% of the academic range), each lasting three weeks, provided by the National Academy for Gifted and Talented Youth (NAGTY);
- RI Mathematics Masterclass Series (chosen as professing characteristics of Type 2): a series of five Saturday-morning masterclasses, each led by different speakers and focused on a different aspect of mathematics, run by a Royal Institution (RI) Masterclass Group at its local university; and
- UKMT Maths Challenge Competitions (chosen as professing characteristics of Types 1 and 3): the Junior and Intermediate Maths Challenge competitions offered by the United Kingdom Mathematics Trust (UKMT), undertaken in a school that was embedding competition problems into its ‘Schemes of Work’ to use for day-to-day teaching and learning with all its students.

Although it was not possible to ‘test’ the framework through initiatives that subscribed, respectively, to each of the prototypes, the goal of this empirical work was, nevertheless, to observe whether, how, and to what extent, the characteristics of the four prototypes were evidenced in practice. With this in mind, each case study explored the ideas and experiences of enrichment, held by the staff organizing the respective initiative and the students participating in it. Data were collected through: analysis of documents relevant to each selected initiative and the particular group or centre (chosen on the basis of recognized good practice) where the enrichment provision being investigated was based; semi-structured interviews (average: 100 minutes) with nineteen key members of staff with responsibilities for organizing and delivering the four enrichment initiatives; as well as end-of-programme, semi-structured interviews (average: 50 minutes) with forty individual student participants, chosen in consultation with teachers and enrichment staff so as to cover, potentially, a range of views and experiences of participating in enrichment. Data analysis and interpretation were further informed by observations, made at the sites of the four initiatives over a more extended period of almost two years.

In order to enhance comparability between and within case studies (comparing enrichment initiatives, as well as ideas and experiences of organizers and participants) and of data collected through different methods, a set of five
underpinning questions was developed from which all data-collection and analytic frames were derived, probing:

- notions and provisions of enrichment;
- goals and outcomes of enrichment;
- audiences targeted by enrichment;
- times and places where enrichment occurs; and
- ideas about whether all learning can be enriched (see Feng, 2005).

The set of underpinning questions provided a structure that permeated the research, right from the initial framing of ideas from the literature to data collection and analysis. During data analysis, themes were raised inductively from the data as well as deductively from the literature, and refined iteratively through constant comparison.

In the following sections, the four chosen initiatives are described in more detail. In each case, the ideas and experiences of the staff and students who organized and participated in the initiative are summarized. The enrichment framework is also applied and assessed in the light of these findings so as to develop a better understanding of how the various prototypes play out in practice and of how the conceptual framework can be applied (including limitations of its application).

4 Case Study 1: NRICH Mathematics Outreach Workshops

4.1 Outline of initiative

NRICH (http://nrich.maths.org/public) began in 1996 as a web-based resource publishing monthly mathematical puzzles aimed at promoting mathematical thinking and problem-solving. By the time of this research, NRICH had developed an extensive web-library of mathematical problems, games and articles, targeted at students aged five and above; its web-presence was active internationally. In addition to its online activities, NRICH personnel also engaged in a large number of school visits in the UK, running mathematics workshops for students as well as in-service training for teachers.

In its Outreach Project, selected Year-10 students (age 14–15) from disadvantaged, inner-city London schools were invited to take part in after-school enrichment classes held weekly throughout the academic year at their local university. There, students worked on a series of NRICH problems, progressively structured in the form of ‘Maths Trails’ (Piggott & Pumfrey, 2005, 2006). Help was at hand from adult facilitators comprising NRICH personnel, mathematics teachers from participating schools, and student volunteers from the University of Cambridge who also acted as mentors for participating students. Teachers from participating schools were also trained by NRICH to deliver some of the classes and to use the same enrichment material (and other NRICH resources) with other children in their school. The aim was to better integrate enrichment with day-to-day mathematics teaching and learning for larger cohorts of students — not just those involved in the Project.

The Outreach Project was funded by a charity that supports educational projects for ‘disadvantaged, disengaged and challenged children’ to help them to ‘recognize and realize their full potential’.

4.2 Summary of findings

The interpretation of enrichment promoted by NRICH emphasized problem-solving experiences that encourage mathematical thinking, supported by specially-designed problems and non-assertive mediation; enrichment was regarded as an integral, pervasive and on-going part of education (characteristic of Type 3). As an Outreach
Project, enrichment was targeted at students living in a ‘disadvantaged’ inner-city area and aimed to ‘raise aspirations for higher education’. Provision was founded on mathematical and social interactions that took place in a university setting (characteristic of Type 4).

This view of enrichment was supported by many experiences reported by members of NRICH, teachers and participants, prominent features of which include: students working together in a collaborative and supportive way, sharing ideas and developing and elucidating their own approaches to solving problems; and teachers being engaged in the process of enquiry alongside students. All parties described the mathematical problems introduced as being possible to tackle in more than one way and capable of being extended to capture emergent lines of enquiry. Solutions were not immediately obvious; careful reasoning was required to reach a solution. One participant described working on the problems as follows:

In [Outreach], when you get [a] problem, you don’t [immediately see] the whole [solution] in front of you. You have to work it out, work at different methods or look at [the problem in] different ways. Then you get an answer after joining two or more solutions together or working out different sections and then joining them together.

Several disparities were also notable. First, although both teachers and students acknowledged that Outreach was supportive of mathematics teaching and learning in school, the enrichment experience was perceived to be different and separate from school mathematics (contrary to Type 3). In particular, teachers and students alike frequently contrasted their experience of teaching and learning mathematics in school with that in Outreach. As far as could be discerned, NRICH’s conception of enrichment as pervasive to all learning remained elusive.

Second, the notion of enrichment as appropriate provision for all students (as in Type 3) was not shared beyond the members of NRICH. Although the teachers acknowledged that all students could benefit from developing the kind of problem-solving skills encouraged by Outreach, they saw the enrichment primarily as provision for high-attaining students (Type 1) and tended to recruit high-attaining students whom they believed would be sufficiently motivated to attend workshops consistently. Similarly, although participants commonly believed that the provision they encountered could benefit all students, their perceptions and experience of Outreach were as a privileged provision for high-attaining students (Type 1), not least because of their awareness of the circumstances surrounding their own selection. This association between enrichment and provision for high-attaining students detracted from NRICH’s intention to engage students from a wide range of attainment. In practice, Outreach came to develop a much stronger emphasis on enriching ‘gifted’ students (as in Type 1) than was intended.

The focus on engaging the ‘disadvantaged’ (as in Type 4) proved similarly challenging. Ironically, as participation after school and over an extended period was a feature of Outreach, the students who persisted in attending tended to be from among the more motivated of their cohort, and not particularly ‘disadvantaged’: many participants were able to draw on support and encouragement from their family, which already had high aspirations for them and valued the opportunities provided. Although a small number of ‘under-motivated’ students had been introduced successfully to Outreach, most teachers believed that motivated students were more likely to benefit from enrichment and either took, or intended to take, motivation into account when selecting students:

This year, [we] just picked students on the basis of their academic achievement. [Some] had the grades but [weren’t] very keen. [It was]
very difficult to bring them back after school. [...] So commitment has to be an important factor [for selection next time].

Participants, too, identified their perceived ‘reliability’ as a key factor in their recruitment. In the end, the aim to engage ‘disadvantaged’ students was undermined by the context: whilst it had been possible to mount enrichment in a disadvantaged area to reach an audience that might not have had access to such opportunities previously, reaching the most disadvantaged children remained a challenge. In this light, participants’ reports about the benefits they had derived from studying in a higher-education environment, which they felt had increased their desire to go to university and interest in studying mathematics, can be understood as being associated as much with outreach (Type 4) as with the development of mathematical talent (Type 1).

Although members of NRICH were aware of the tensions described and sought to persuade both teachers and participants to their perspective, the teachers’ interests in, and view of, enrichment as being more closely aligned with the development of talent (Type 1) ultimately dominated proceedings. A powerful factor in this negotiation appeared to be the more extended contact the teachers had with participants (in Outreach and in school) and the teachers’ role in selecting participants; both provided opportunities through which the teachers could shape expectations. In the end, Outreach was gradually prompted to adopt a stance in the development of mathematical talent (Type 1).

5 Case Study 2: NAGTY Mathematics Summer Schools

5.1 Outline of initiative

NAGTY was established by the UK Department for Education and Skills (DfES) in 2002 and continued in operation until 2007. Its remit was to co-ordinate the development and delivery of education for the top 5% of the population, up to the age of 19. NAGTY’s Summer Schools — its ‘flagship’ programme — encapsulated its ‘wider schooling pedagogy’ (Persaud & Roberts, 2006): to supplement provision available for gifted students locally and regionally, and to enable participants to work alongside other gifted students and subject experts in higher-education institutions across England.

In the three-week residential programme visited by this research, students aged 11–14 and 14–16 typically engaged in 6–7 hours of structured and independent academic study on working days, combined with leisure programmes in the evenings and at weekends. Courses were taught primarily by academics from the host institution, with help from local teachers and teaching assistants (mostly graduates and postgraduate-students from the host department) (Office for Standards in Education, 2004, 2005). Admission was selective and two-fold: only NAGTY ‘members’ — students already admitted to NAGTY on the basis of their academic performance or potential — were eligible to apply.

Core funding for NAGTY programmes came from DfES, though NAGTY also sought commercial sponsorship for its programmes. For the Summer-Schools, participants and their schools also paid a share towards the (subsidized) cost of the course. Bursaries were available to support participants from poorer households.

5.2 Summary of findings

The predominant enrichment concern espoused by, and governing, the Summer Schools was the development of talent (Type 1). In this respect, organizers’
conceptions and intentions were closely matched by participants’ perceptions and experience; many commonalities can be found from the reports of both.

First, in common with all NAGTY provisions, the Summer Schools were targeted solely at students performing in the top 5% academically. The two-fold admission process, whereby students actively selected and applied for courses, was identified by NAGTY as one of the factors that underpinned the initiative’s success. Similarly, participants thought that the Summer Schools were to be of primary benefit to high-attaining students. Almost all participants had devoted some time to researching available options, demonstrating considerable initiative and commitment in becoming involved. As participants observed, this investment might have helped to ensure that a positive experience was had by all.

Second, NAGTY had a generic focus on cultivating ‘gifted’ students: students’ personal and social development was given equal emphasis to academic enrichment. Participants also chose to attend Summer Schools as much for the social experience as for the mathematics: while the latter was a subject students enjoyed and served as a focus for bringing like-minded students together, the resulting social and intellectual interactions were at least as important to them; academic and social aspects of the Summer Schools were appreciated equally. As one participant noted:  

We’re always working in groups [in the classes and in our evening social activities]. That’s good because you get to socialize as well as learn from other people.

Third, organizers regarded the extended period for which they were in intensive contact with participants as a distinctive and important aspect of the Summer Schools, allowing them to develop concepts and to present material in a more measured way. Whilst there was no evidence that participants appreciated the extended contact as a distinctive part of provision, they did report being able to take their time to enjoy the various activities and being able to develop mathematical ideas through series of connected activities. Participants reported finding this more joined-up presentation of mathematics helpful, as it demonstrated to them how mathematical ideas from different areas could be synthesized flexibly.

Finally, organizers drew on their own perceptions and excitement about mathematics as mathematicians, and on their experience of learning mathematics as gifted children, to formulate provision. One organizer expressed this as follows:

I’d [like students to gain] a proper understanding of what maths is, [because] what I’ve learnt in school is nothing like what maths [is about]. Most people [think it’s about] completely separate, random topics [...] and that’s wholly false. [Maths] is really a massively interconnected whole.

Participants were introduced to powerful concepts from degree- and research-level mathematics (as might also be identified with Type 2), including concepts which they were not expected to understand. This conscious departure from school mathematics was intended to provide participants with a ‘real’ and positive experience of mathematics, to introduce applications of mathematics in surprising contexts and stimulating ideas not explored in school, and to induct participants into working in an environment of genuine challenge. In this process, organizers perceived themselves as ‘guides’ to facilitate and encourage the participants’ own exploration.

Participants’ accounts of their experience mirrored this intention: the substantive material was found to be interesting and more challenging than school mathematics; participants also reported having greater freedom and responsibility for doing their own work. As a result, the majority of participants felt that their interest in
mathematics and in studying mathematics had increased and that they had become more aware of the pervasiveness of mathematics and its applications.

Such similarities aside, there were some disparities in the significance organizers and students attached to being in a higher-education institution. The involvement of higher-education institutions was central to NAGTY’s enrichment model (Campbell, Eyre, Muijs, Neeland, & Robinson, 2004; Eyre, 2004): besides enabling participants to work with academic experts and providing them with inspiring role models, gaining experience of university life by virtue of being in a higher-education environment was also thought to contribute to the success of the Summer Schools. Very few participants, however, had reported associating any of their experiences specifically with the university setting, though it was not clear whether this might have been because they had not then had any direct knowledge of higher education.

6 Case Study 3: RI Mathematics Masterclass Series

6.1 Outline of initiative

The RI has a long history in the UK of engaging the general public in scientific discourse through popular seminars by eminent scientists, dating back to the public lectures given by its earliest scientists, Sir Humphry Davy and Michael Faraday. Its Masterclasses for primary- and secondary-school students grew from the 1978 Christmas Lectures given by Sir Christopher Zeeman. By the time of this research, the RI had become a reference point for a network of autonomous Masterclass Groups (consisting mainly of volunteers based in universities, schools and local authorities) which, between them, were running around sixty Masterclass series across over forty locations in the UK (Kathotia, 2006).

Provision offered by the Group visited by this research was typical of ‘Secondary Masterclasses’: students in Years 8 or 9 (age 12–13), recruited on the basis of teacher recommendation, were invited to attend a series of five two-and-a-half-hour Saturday-morning workshops at a nearby university, led by university lecturers and mathematics educators. The workshops typically consisted of periods of formal teaching, interspersed with opportunities for students to explore the subject on their own or in small groups. The topics presented were selected by the Masterclass speakers to reflect their own interests, resulting in a range of mathematical topics being discussed over the course of a series.

Funding for the Masterclasses was raised by the Group through calls to local businesses and university sources on a needs basis once every few years. Many speakers and volunteers contributed for free; some supporters contributed in kind (e.g. through providing accommodation and facilities free of charge).

6.2 Summary of findings

Organizers and participants agreed on many aspects of the Masterclasses. Both perceived the provision as one that introduced students to a variety of mathematical topics and applications not normally studied in school (characteristic of Type 2). Participants found this experience stimulating and enjoyable; many also reported developing an improved perception of the subject and a greater insight into mathematics beyond that which is normally taught in school. Although the mathematical topics featured did not appeal to students equally, participants liked the variety of mathematics represented because this enabled them to find something they enjoyed. One participant summarized this as follows:
They’ve chosen the topics so that they have something of everything in the series. Somebody who might not have enjoyed one session would hopefully have liked other ones better.

This variety is also recognized by organizers as a strength of the Masterclass format.

Second, both organizers and participants saw the Masterclasses as an opportunity for students to meet and work with fellow participants who were also interested in mathematics and of a similar level of attainment. Collaborative work, discussion, and learning from fellow participants were experiences commonly identified and valued by both. These interactions as well as opportunities to explore mathematics and to work on challenging problems were key to the stimulating experience reported by participants. For both organizers and participants, the Masterclasses supplemented mathematics learning in school, but were, to a large extent, unconnected to it.

Some constraints of the Masterclass format were recognized and experienced by both organizers and participants. These stemmed primarily from the brevity of each Masterclass and of the series as a whole: the lack of familiarity and limited contact with speakers and fellow participants hampered exchange of ideas, limited coverage of material, and reduced opportunities for participants to question and to take initiative in their own learning. In addition, only those participants who could (or whose family could) accommodate the commitment on Saturdays could attend.

Several differences between organizers’ and participants’ perspectives were also notable. First, whereas organizers envisioned the Masterclasses as a forum for mathematical exchange where enthusiastic speakers and students would engage in mutually-satisfying interactions, the stimulating exchanges identified by participants only involved their fellow students. Speakers made few appearances in participants’ accounts, and then, only in relation to how well they had introduced their selected topic and engaged students in activities. Given the reported barriers which hampered both social and intellectual exchange between participants and between participants and speakers, it is doubtful whether participants saw themselves as a part of a ‘mathematical community’ as organizers had hoped. Participants also did not report any improvements in confidence or in their image of themselves in relation to mathematics.

Finally, the vast majority of participants saw the Masterclasses as a provision that was catered to their needs as ‘mathematically-able’ students (consistent with Type 1); for some, attending Masterclasses was a reward for good achievement in school. Indeed, organizers’ experience suggested that teachers, when asked to recommend participants, frequently nominated their highest-attaining students for the limited Masterclass places (consistent with Type 1), with the result that Masterclass cohorts were commonly higher-attaining than classes in schools. Such actions also found support among organizers at the Group level, who saw a role for the Masterclasses in engaging ‘able’ students who might not otherwise have regular access to stimulating mathematical experiences in school. As one organizer commented:

It’s really about giving every child the opportunity to have a challenging and enjoyable curriculum. [It’s] giving kids [whose] normal circumstances are not conducive to that kind of [experience] opportunity to do mathematics [in] a different set-up. [...] I think that’s fair.

As such, Masterclass provision was formulated with ‘able’ students in mind (as in Type 1), both because organizers at the Group level viewed the Masterclasses as being such a provision, and in order to cater for the high-attaining participants attending Masterclasses, selected independently by their teachers. The institutional vision of the RI and its expressed intention to engage a wider range of students through its Masterclasses (as would be consistent with Type 2) received only limited
support at the Group level and among participants, despite being promoted actively by Masterclass organizers at the RI level. As organizers at both RI and Group level recognized, Masterclass Groups were autonomous of the RI and the power to shape provision ultimately rested with them.

7 Case Study 4: UKMT Maths Challenge Competitions

7.1 Outline of initiative

Formed in 1996, the UKMT brought together several UK mathematics competitions (which had previously been organized by separate bodies) under its Maths Challenges (United Kingdom Mathematics Trust, 2004). With a reach of over half a million students, participating from over four thousand schools at the time of this research, the Challenges were the largest mathematics competitions in the UK. At the Senior level, the Challenges fed into the training and selection of exceptionally mathematically-gifted young people to represent the UK in the International Mathematical Olympiad (IMO). This research, however, focused on the Junior and Intermediate Challenges in which the vast majority of secondary-school students who participated in UKMT initiatives competed.

Presented as multiple-choice papers to be taken under examination conditions on specified dates in participants’ schools, the Junior and Intermediate Challenges consisted of twenty-five problems, of which the first fifteen were more accessible. Certificates were awarded to the top 40% of participants: Gold, Silver, Bronze, and ‘Best in School’ according to performance. Only a very small number of students consisting of the most successful competitors were invited for further rounds of competition and mentoring.

A charge was levied on schools for Challenge ‘entries’. As an educational charity, the UKMT also received sponsorship from companies and generated income through publishing mathematical resources. Many supporters contributed to UKMT activities on a voluntary basis.

7.2 Summary of findings

The most salient aspect of the Challenges identified by both organizers and participants was the quality of the questions, which was consistently praised. From the organizers’ perspective, the problems were deliberately formulated so as not to be amenable to routine application of rehearsed techniques; instead, their goal was to stimulate mathematical thinking beyond that normally demanded by standard textbook questions, while drawing on mathematics studied in school. This aspect of the Challenges was welcomed by teachers and students, who typically perceived the material in the same way. In so far as it was possible to stimulate students through brief engagement with the Maths-Challenge material during competitions (which, despite the school’s efforts to embed the problems into day-to-day teaching and learning, was the primary way the students saw themselves as engaging with the material), students confirmed that the problems had challenged their thinking and had made them realize that mathematics is more than that which they learn in school (drawing only limitedly on Type 3). One student described her experience as follows:

I didn’t know there [could be] that much fun working through these problems. I think that’s why I found [the Challenge] enriching. It was just a different experience. It’s weird: [the questions work] your brain so hard but still you find it quite fun. [...] It’s just using your mind so much more. [When] you open the textbook, you know what topic [the
questions are] on. But you don’t with the Maths Challenge. [That’s] what makes it more exciting.

The greatest difference between organizers’ conceptions of the Challenges and the way the Challenges were perceived and experienced by participants lay in the competitive element. Although organizers were well-aware of the limitations of the competitive format, its elitist connotations were not given emphasis. Instead, the focus was on engaging large numbers of students from amongst those with reasonable competency; the Challenges, it was believed, could be undertaken without additional learning or preparation. Teachers, too, did not perceive any difference in benefits as a consequence of students achieving different results. For students, however, the competitive element was central to their Maths-Challenge experience. The latter was, in turn, mediated by more familiar school experiences of assessment and of preparing for examinations. Indeed, students spontaneously and consistently referred to the Challenges as ‘tests’ or ‘exams’ — notions reinforced, perhaps, by the way the Challenges were administered in school and the compliance some participants felt was expected of them in taking part. In the light of comparisons and depending on results, students who had ‘done well’ took pride in their achievement. As one student commented:

[What I particularly like is getting] a certificate at the end — that's good! And you can compare your scores with your friends. [...] For a certificate, you feel like you’ve done well at something, [and] most of the time, I do better than my friends, so it feels good.

Some expressed disappointment in their own performance. Given this reaction, participants valued opportunities to ‘practise’ solving problems from past competitions. Such deliberate preparation, which mirrored revision for school examinations, was thought to enhance performance, and hence, the overall enrichment experience; those performing well in the Challenges were thought to benefit more from taking part.

Several complementarities between organizers’ and participants’ perspectives were also notable. First, organizers in the UKMT wanted to increase the uptake of the Challenges as a matter of priority. Many students, meanwhile, were participating because their school had entered them, not because they had any wish to be involved. As one student noted:

I didn’t particularly want to do [the Maths Challenge]. It’s just something all the Set 1s do here in this school. I don’t really mind.

This differed from the other enrichment initiatives studied, which, although not reaching as large a number of students, recruited participants who often exercised considerable enterprise and commitment in becoming involved or had a ready interest in mathematics. In one sense, it may be argued that the Challenges were succeeding in their objective of reaching students who might not self-identify as ‘good mathematicians’ or naturally put themselves forward for mathematics-related opportunities. How such students might perceive their enrichment experience and be affected by it, however, had not been pursued further.

Second, as a step in the identification and cultivation of mathematical talent, organizers saw the Challenges as opportunities for students to demonstrate, discover, and celebrate their capability (as might be strongly associated with Type 1). Indeed, the outcomes of the Challenges did draw students’ attention to their own mathematics learning and achievement. As another ‘test’, however, the Challenges also made apparent differences in performance in ways that might prove uncomfortable for some participants.
Finally, the Challenges were less dependent on the mediation of teachers and the availability of school resources than some other forms of enrichment. From the organizers' perspective, the Challenges therefore offered distinctive opportunities in support of students' learning. This reduced dependency was echoed in participants' perceptions: the primary way that students reported of being engaged in Maths-Challenge material was through participating in the (one-off) competitions; teachers were not perceived by their students as having any ownership or responsibility in this process. This differed from the organizers' aspiration that teachers should play a crucial role in embedding and furthering enrichment beyond minimally exposing students to the Challenges (as might be identified with Type 3); there was certainly no intention of excluding teachers from the enrichment process — quite the opposite.

8 (Re)Conceptualizing mathematics enrichment

8.1 Enrichment is multi-dimensional and draws variously on four prototypes

Four prototypes were initially derived from the literature to form a first framework for understanding mathematics enrichment. This framework proved to be a good initial heuristic for characterizing enrichment initiatives and for understanding enrichment in the abstract: evidence from the four case studies verified that enrichment is indeed multi-dimensional and comprises (at least) the four types distinguished originally. In practice, however, initiatives may not exhibit all the characteristics of any one prototype and may draw on more than one prototype.

8.2 Enrichment is a product of interactions, influenced by context and conceptualizations of ‘mathematics’ and ‘learning’

Organizers’ enrichment conceptions and intentions have a powerful influence on enrichment ethos and provision. In all four case studies, there were many similarities between organizers’ reported conceptions and participants’ reported experience. That said, the case studies also illustrated that the eventual form taken by an enrichment initiative is a product of interactions between the people and institutions involved. Different organizers may have different interests, motivations, conceptions of ‘mathematics’ and ‘learning’, and values (including different ways of ‘valuing’ mathematics); funders, intermediaries and end-users’, too, may have different interests, expectations, conceptions of ‘mathematics’ and ‘learning’, and values that bear on enrichment, sometimes in a way that may not be consistent with organizers’ thinking and stated intentions.; Initial conceptions and intentions are moderated as they are disseminated through the enrichment structure.

Moreover, different programme formats can create different possibilities and impose different constraints on the enrichment outcomes that could be achieved. Participants’ perceptions and experience may also be further mediated by the enrichment context and ‘everyday’ experiences against which enrichment may be understood. These mediating influences may also give rise to unintended outcomes.

Speculatively, the moderation of enrichment conceptions and intentions during dissemination and the mediating influences affecting participants’ perceptions and experience may be more pronounced in larger-scale initiatives, both because such initiatives are likely to involve more individuals (whether in an organizational, mediational, or delivery capacity), and because those involved are likely to be engaged in a more complex socio-politico-ideological infrastructure. Contrasting in these respects, smaller-scale initiatives may have a comparatively tighter, yet more fluid, ‘focus’ — even if this ‘focus’ may not appear to be as well theorized or
explicitly defined to an observer. As such, smaller-scale initiatives may be able to carry out, and perhaps even change, its goals in a relatively more straightforward manner. Having only investigated larger-scale initiatives, this study is limited in the light it can throw on these (and other) issues (where the scale of the programme might be a dependent factor).

In sum, the interactions and contextual factors highlighted by the case studies are not captured by the framework, and indeed, cannot be captured fully by any conceptual framework alone. By giving explicit examples of how various interactions and contextual factors have shaped actual enrichment initiatives, and drawing attention to the dynamic interplay between contextual factors, the four case studies form an important supplement to the framework by informing both how the framework could be applied as well as the limitations of its application.

8.3 Unequal interplay between prototypes of enrichment

Further to the interactions described, ideas from the four prototypes do not play out unproblematically or on an equal basis.

The development of talent (Type 1) has long been the dominant discourse within the enrichment literature. Available evidence suggests that developing mathematical talent has retained a strong position within enrichment enterprises and is more widely associated with enrichment than the other three prototypes. In observed interplay with other prototypes, the development of talent had often emerged as dominant, even in initiatives where it was not a centrally promoted part of the provision. For example: in Outreach, the enrichment of underprivileged children (Type 4) had become identified with provision for mathematically-high-attaining children in broadly underprivileged areas; in the Challenges, the engagement of the wider student population in enhanced forms of mathematical thinking (Type 3) competed and was in tension with the identification and cultivation of children with exceptional mathematical talent; in the Masterclasses, the popular contextualization of mathematics (Type 2) was focused on an audience that had been largely selected on the basis of their mathematical attainment. This may be another reflection of the interplay, highlighted above, between the conceptions of ‘mathematics’ and ‘learning’ of those involved in enrichment initiatives.

To gain a fuller understanding of enrichment, the proposed framework must be applied contextually, giving due attention to the unequal dynamics in the interplay between the various prototypes of enrichment and the conceptions of ‘mathematics’ and ‘learning’ that underpin them.

8.4 Distinguishing aspects of enrichment as enhancing proficiency and learning processes

Over the course of study, this research has encountered some initiatives (e.g. the UKMT Maths Challenges) which had a broad aim of developing mathematical skills and understanding. NRICH, however, was distinctive in having a well-elucidated pedagogy for developing students’ mathematical thinking at the heart of its provision. Whilst both forms of enrichment were previously characterized as ‘enhancing mathematical proficiency and learning processes’ (Type 3), case studies into the Challenges and the work of NRICH have shown that the two were, in fact, quite different: the former exposed children to concisely-formulated, ‘non-routine’ problems intended to promote and encourage more ‘lateral’ mathematical thinking; the latter inducted and supported children through a series of problems individually designed and collectively structured to enhance particular aspects of problem-solving (e.g. using systematic strategies, generalizing mathematical results). To
recognize these two facets, Type 3 may be subdivided into the promotion of mathematical proficiency and understanding (Type 3A) and the enhancement of mathematical processes through supported activities (Type 3B).

9 Conclusion

This study represents a first effort in conceptualizing mathematics enrichment, focused on current conceptions and practices in the UK. Through synthesizing existing ideas from the literature and new empirical evidence from four initiatives, this study has shown that enrichment is a multi-dimensional concept that draws variously and dynamically on four prototypes and is a product of context-dependent interactions between the people and institutions involved. The resulting forms of enrichment differ not only in conception and delivery, but also in the nature and process of negotiation that shapes provision. These findings have several implications for policy, practice and research.

First, the complexities of enrichment should be recognized, both in conception and implementation. The meaning of enrichment cannot be assumed; closer attention should be paid to the context-dependent interactions associated with different forms of enrichment in research and when reporting enrichment efforts. This will facilitate the sharing of enrichment ideas and practices among practitioners and researchers and help to convey clearer messages to stakeholders (e.g. funders and policymakers).

Second, since enrichment is a product of interactions between the people and institutions involved, the power to shape enrichment is shared in the same way. On one hand, anyone involved can initiate change. Individual creativity can flourish; individual effort can make a big strategic difference. On the other hand, gaining wider acceptance for ideas and making lasting changes to practice is more difficult to accomplish. In order for any concept to become an enduring feature of enrichment, many others must be brought onboard and motivated to act upon the concept. Long-held perceptions — even prejudices — may have to be reshaped. Systemic change, then, is necessarily a long-term process; the dissemination of enrichment thinking and good practice is necessarily an ongoing process.

Finally, and most fundamentally, the endurance and proliferation of mathematics enrichment raises pressing questions about the adequacy of the school mathematics curriculum and the engagement of young people in mathematics. Why is school mathematics perceived as being so ‘impoverished’ that individuals felt the need to initiate enrichment programmes outside the formal system of education? Is there anything that can or should be done within mainstream school-mathematics provision? Why does enrichment in mathematics appear to have such a strong association with the development of talent (Type 1) in the minds of so many UK-based practitioners and participants? Is the UK ‘peculiar’ in this respect? How widespread is this phenomenon? How should ‘mathematics’ and ‘learning’ be conceptualized in the school curriculum and in enrichment? Is it ‘right’ to offer enrichment when this may be accessible only to a fraction of young people? Is it ‘right’ to try to engage students who ‘have no interest in mathematics’ through enrichment? Such questions should be tackled as a matter of priority by policymakers in collaboration with the mathematics-education community.

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